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Class - 9th Rajpur, Balrampur.
subject :- Maths

Q.①:- write and remember table upto 20.

Q.②:-Define :-

(i) Rational Number and it's type.
(ii) Irrational Number
(iii) Polynomials ~~and it~~ and Zero Polynomial
(iv) Remainder Theorem
(v) Abscissa & ordinate.
(vi) Factor Theorem.

Q.③:- Solve. Exercise:- 1.2; 1.3 and 1.5 from Chap. 01.
and Exercise!- 2.2; 2.4 and 2.5 from Chap. 02

Q ④:- Explain Algebric Identities with example (Chap 02)

Q.⑤:- project work.

(i) Draw a Cartesian plane on Chart paper and Explain it's quadrants and axis.
(ii) write the Statement of Remainder Theorem and prove it.
(iii) State and prove Factor Theorem.

located.

EXERCISE 1.2

1. State whether the following statements are true or false. Justify your answers.

   (i) Every irrational number is a real number.

   (ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural numbe

   (iii) Every real number is an irrational number.

2. Are the square roots of all positive integers irrational? If not, give an example of th square root of a number that is a rational number.

3. Show how $\sqrt{5}$ can be represented on the number line.

EXERCISE 1.3

1. Write the following in decimal form and say what kind of decimal expansion each has :

   (i) $\frac{36}{100}$ (ii) $\frac{1}{11}$ (iii) $4\frac{1}{8}$

   (iv) $\frac{3}{13}$ (v) $\frac{2}{11}$ (vi) $\frac{329}{400}$

2. You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division? If so, how?

   [**Hint :** Study the remainders while finding the value of $\frac{1}{7}$ carefully.]

3. Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

   (i) $0.\overline{6}$ (ii) $0.4\overline{7}$ (iii) $0.\overline{001}$

4. Express 0.99999 .... in the form $\frac{p}{q}$. Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

6. Look at several examples of rational numbers in the form $\frac{p}{q}$ $(q \neq 0)$, where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

7. Write three numbers whose decimal expansions are non-terminating non-recurring.

8. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

9. Classify the following numbers as rational or irrational :

   (i) $\sqrt{23}$ (ii) $\sqrt{225}$ (iii) 0.3796

   (iv) 7.478478... (v) 1.101001000100001...

EXERCISE 1.5

1. Classify the following numbers as rational or irrational:

   (i) $2 - \sqrt{5}$ (ii) $\left(3 + \sqrt{23}\right) - \sqrt{23}$ (iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

   (iv) $\frac{1}{\sqrt{2}}$ (v) 2π

2. Simplify each of the following expressions:

   (i) $\left(3 + \sqrt{3}\right)\left(2 + \sqrt{2}\right)$ (ii) $\left(3 + \sqrt{3}\right)\left(3 - \sqrt{3}\right)$

   (iii) $\left(\sqrt{5} + \sqrt{2}\right)^2$ (iv) $\left(\sqrt{5} - \sqrt{2}\right)\left(\sqrt{5} + \sqrt{2}\right)$

3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi = \frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

4. Represent $\sqrt{9.3}$ on the number line.

5. Rationalise the denominators of the following:

   (i) $\frac{1}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{7} - \sqrt{6}}$

   (iii) $\frac{1}{\sqrt{5} + \sqrt{2}}$ (iv) $\frac{1}{\sqrt{7} - 2}$

Now, $2x + 1 = 0$ gives us $x = -\frac{1}{2}$

So, $-\frac{1}{2}$ is a zero of the polynomial $2x + 1$.

Now, if $p(x) = ax + b$, $a \neq 0$, is a linear polynomial, how can we find a zero of $p(x)$? Example 4 may have given you some idea. Finding a zero of the polynomial $p(x)$, amounts to solving the polynomial equation $p(x) = 0$.

Now, $p(x) = 0$ means $ax + b = 0, a \neq 0$

So, $ax = -b$

i.e., $x = -\frac{b}{a}$.

So, $x = -\frac{b}{a}$ is the only zero of $p(x)$, i.e., a *linear polynomial has one and only one zero.*

Now we can say that 1 is *the* zero of $x - 1$, and -2 is *the* zero of $x + 2$.

Example 5 : Verify whether 2 and 0 are zeroes of the polynomial $x^2 - 2x$.

Solution : Let $p(x) = x^2 - 2x$

Then $p(2) = 2^2 - 4 = 4 - 4 = 0$

and $p(0) = 0 - 0 = 0$

Hence, 2 and 0 are both zeroes of the polynomial $x^2 - 2x$.

Let us now list our observations:

(i) A zero of a polynomial need not be 0.

(ii) 0 may be a zero of a polynomial.

(iii) Every linear polynomial has one and only one zero.

(iv) A polynomial can have more than one zero.

EXERCISE 2.2

1. Find the value of the polynomial $5x - 4x^2 + 3$ at
   (i) $x = 0$ (ii) $x = -1$ (iii) $x = 2$
2. Find $p(0)$, $p(1)$ and $p(2)$ for each of the following polynomials:
   (i) $p(y) = y^2 - y + 1$ (ii) $p(t) = 2 + t + 2t^2 - t^3$
   (iii) $p(x) = x^3$ (iv) $p(x) = (x - 1)(x + 1)$

3. Verify whether the following are zeroes of the polynomial, indicated against them.
   (i) $p(x) = 3x + 1$, $x = -\frac{1}{3}$ (ii) $p(x) = 5x - \pi$, $x = \frac{4}{5}$
   (iii) $p(x) = x^2 - 1$, $x = 1, -1$ (iv) $p(x) = (x + 1)(x - 2)$, $x = -1, 2$
   (v) $p(x) = x^2$, $x = 0$ (vi) $p(x) = lx + m$, $x = -\frac{m}{l}$
   (vii) $p(x) = 3x^2 - 1$, $x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$ (viii) $p(x) = 2x + 1$, $x = \frac{1}{2}$
4. Find the zero of the polynomial in each of the following cases:
   (i) $p(x) = x + 5$ (ii) $p(x) = x - 5$ (iii) $p(x) = 2x + 5$
   (iv) $p(x) = 3x - 2$ (v) $p(x) = 3x$ (vi) $p(x) = ax$, $a \neq 0$
   (vii) $p(x) = cx + d$, $c \neq 0$, c, d are real numbers.

2.4 Remainder Theorem

Let us consider two numbers 15 and 6. You know that when we divide 15 by 6, we get the quotient 2 and remainder 3. Do you remember how this fact is expressed? We write 15 as

$$15 = (6 \times 2) + 3$$

We observe that the *remainder* 3 is less than the *divisor* 6. Similarly, if we divide 12 by 6, we get

$$12 = (6 \times 2) + 0$$

What is the remainder here? Here the remainder is 0, and we say that 6 is a *factor* of 12 or 12 is a *multiple* of 6.

Now, the question is: can we divide one polynomial by another? To start with, let us try and do this when the divisor is a monomial. So, let us divide the polynomial $2x^3 + x^2 + x$ by the monomial x.

We have $(2x^3 + x^2 + x) \div x = \frac{2x^3}{x} + \frac{x^2}{x} + \frac{x}{x}$

$= 2x^2 + x + 1$

In fact, you may have noticed that x is common to each term of $2x^3 + x^2 + x$. So we can write $2x^3 + x^2 + x$ as $x(2x^2 + x + 1)$.

We say that x and $2x^2 + x + 1$ are *factors* of $2x^3 + x^2 + x$, and $2x^3 + x^2 + x$ is a *multiple* of x as well as a multiple of $2x^2 + x + 1$.

EXERCISE 2.4

1. Determine which of the following polynomials has $(x + 1)$ a factor :

   (i) $x^3 + x^2 + x + 1$

   (ii) $x^4 + x^3 + x^2 + x + 1$

   (iii) $x^4 + 3x^3 + 3x^2 + x + 1$

   (iv) $x^3 - x^2 - \left(2 + \sqrt{2}\right)x + \sqrt{2}$

2. Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = 2x^3 + x^2 - 2x - 1, g(x) = x + 1$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1, g(x) = x + 2$

(iii) $p(x) = x^3 - 4x^2 + x + 6, g(x) = x - 3$

3. Find the value of k, if $x - 1$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = x^2 + x + k$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}\,x + 1$

(iv) $p(x) = kx^2 - 3x + k$

4. Factorise :

(i) $12x^2 - 7x + 1$

(ii) $2x^2 + 7x + 3$

(iii) $6x^2 + 5x - 6$

(iv) $3x^2 - x - 4$

5. Factorise :

(i) $x^3 - 2x^2 - x + 2$

(ii) $x^3 - 3x^2 - 9x - 5$

(iii) $x^3 + 13x^2 + 32x + 20$

(iv) $2y^3 + y^2 - 2y - 1$

Example 24 : Factorise $8x^3 + 27y^3 + 36x^2y + 54xy^2$

Solution : The given expression can be written as

$$(2x)^3 + (3y)^3 + 3(4x^2)(3y) + 3(2x)(9y^2)$$
$$= (2x)^3 + (3y)^3 + 3(2x)^2(3y) + 3(2x)(3y)^2$$
$$= (2x + 3y)^3 \quad \text{(Using Identity VI)}$$
$$= (2x + 3y)(2x + 3y)(2x + 3y)$$

Now consider $(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

On expanding, we get the product as

$$x(x^2 + y^2 + z^2 - xy - yz - zx) + y(x^2 + y^2 + z^2 - xy - yz - zx)$$
$$+ z(x^2 + y^2 + z^2 - xy - yz - zx) = x^3 + xy^2 + xz^2 - x^2y - xyz - zx^2 + x^2y$$
$$+ y^3 + yz^2 - xy^2 - y^2z - xyz + x^2z + y^2z + z^3 - xyz - yz^2 - xz^2$$
$$= x^3 + y^3 + z^3 - 3xyz \quad \text{(On simplification)}$$

So, we obtain the following identity:

Identity VIII : $\mathbf{x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)}$

Example 25 : Factorise : $8x^3 + y^3 + 27z^3 - 18xyz$

Solution : Here, we have

$$8x^3 + y^3 + 27z^3 - 18xyz$$
$$= (2x)^3 + y^3 + (3z)^3 - 3(2x)(y)(3z)$$
$$= (2x + y + 3z)[(2x)^2 + y^2 + (3z)^2 - (2x)(y) - (y)(3z) - (2x)(3z)]$$
$$= (2x + y + 3z)(4x^2 + y^2 + 9z^2 - 2xy - 3yz - 6xz)$$

EXERCISE 2.5

1. Use suitable identities to find the following products:
   (i) $(x+4)(x+10)$ (ii) $(x+8)(x-10)$ (iii) $(3x+4)(3x-5)$
   (iv) $(y^2+\frac{3}{2})(y^2-\frac{3}{2})$ (v) $(3-2x)(3+2x)$
2. Evaluate the following products without multiplying directly:
   (i) 103×107 (ii) 95×96 (iii) 104×96
3. Factorise the following using appropriate identities:
   (i) $9x^2+6xy+y^2$ (ii) $4y^2-4y+1$ (iii) $x^2-\frac{y^2}{100}$

4. Expand each of the following, using suitable identities:
   (i) $(x+2y+4z)^2$ (ii) $(2x-y+z)^2$ (iii) $(-2x+3y+2z)^2$
   (iv) $(3a-7b-c)^2$ (v) $(-2x+5y-3z)^2$ (vi) $\left[\frac{1}{4}a-\frac{1}{2}b+1\right]^2$
5. Factorise:
   (i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$
   (ii) $2x^2+y^2+8z^2-2\sqrt{2}\,xy+4\sqrt{2}\,yz-8xz$
6. Write the following cubes in expanded form:
   (i) $(2x+1)^3$ (ii) $(2a-3b)^3$ (iii) $\left[\frac{3}{2}x+1\right]^3$ (iv) $\left[x-\frac{2}{3}y\right]^3$
7. Evaluate the following using suitable identities:
   (i) $(99)^3$ (ii) $(102)^3$ (iii) $(998)^3$
8. Factorise each of the following:
   (i) $8a^3+b^3+12a^2b+6ab^2$ (ii) $8a^3-b^3-12a^2b+6ab^2$
   (iii) $27-125a^3-135a+225a^2$ (iv) $64a^3-27b^3-144a^2b+108ab^2$
   (v) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$
9. Verify : (i) $x^3+y^3=(x+y)(x^2-xy+y^2)$ (ii) $x^3-y^3=(x-y)(x^2+xy+y^2)$
10. Factorise each of the following:
    (i) $27y^3+125z^3$ (ii) $64m^3-343n^3$
    [**Hint :** See Question 9.]
11. Factorise : $27x^3+y^3+z^3-9xyz$
12. Verify that $x^3+y^3+z^3-3xyz=\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$
13. If $x+y+z=0$, show that $x^3+y^3+z^3=3xyz$.
14. Without actually calculating the cubes, find the value of each of the following:
    (i) $(-12)^3+(7)^3+(5)^3$
    (ii) $(28)^3+(-15)^3+(-13)^3$
15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

Area : $25a^2-35a+12$	Area : $35y^2+13y-12$
(i)	(ii)