PRACTICE PAPER-IV

SUB:- MATHEMATICS

CLASS - XII

Time : 3 hrs | Max. Marks : 80

General Instructions

1. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks.
2. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions
3. Both Part A and Part B have choices.

PART – A

1. It consists of two sections- I and II.
2. Section I comprises of 16 Very Short Answer Type Questions.
3. Section II contains two case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

PART – B

1. It consists of three sections- III, IV and V.
2. Section III comprises of 10 questions of 2 marks each.
3. Section IV comprises of 7 questions of 3 marks each.
4. Section V comprises of 3 questions of 5 marks each.
5. Internal choice is provided in 3 questions of Section –III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

PART A

Section I

All questions are compulsory. In case of internal choices attempt any one.

1. If $R=\{(3,3),(6,6),(9,9),(12,12),(6,12),(3,9),(3,12),(3,6)\}$ is a relation on the set $A=\{3,6,9,12\}$. Then, show that the relation is reflexive and transitive but not symmetric.
2. A function $f:A\rightarrow B$ is such that $f(x_1)=f(x_2)\Rightarrow x_1=x$, then find whether f is one-one or onto.
3. If the set A contains 5 elements and the set B contains 6 elements, then find the number of one-one and onto mapping from A to B is

* You are advised to attempt this sample paper without referring the solutions given here. However, cross check your solutions with the solutions given at the end after you completed the paper.

4. If A and B are symmetric matrices, then show that $AB - BA$ is a skew-symmetric matrix.

Or If the determinant of matrix A of order 3×3 is of value 4. Then, find the value of $|\text{adj}\ (\text{adj}A)|$.

5. If $[x, 1]\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = 0$, find x.

6. Evaluate $\begin{vmatrix} x^3 & x^2 + x + 1 \\ x - 1 & 1 \end{vmatrix}$

7. Find the slope of normal at the point (1, 1) on the curve $2y + x^2 = 3$.

Or

Find the equation of the normal to the curve $y = \sin x$ at (0, 0).

8. If $y = x^{\sin x}$, then find $\frac{dy}{dx}$.

Or

If $y = \sqrt{a^2 - x^2}$, then prove that $y\frac{dy}{dx} + x = 0$.

9. Find $\int_2^3 \frac{dx}{1 - x^2}$

Or

Find $\int_1^2 \frac{dx}{x\sqrt{x^2 - 1}}$

10. Find all the vectors of magnitude $10\sqrt{3}$ that are perpendicular to the plane of $\hat{i} + 2\hat{j} + \hat{k}$ and $-\hat{i} + 3\hat{j} + 4\hat{k}$.

11. If $\vec{a}$ is a unit vector such that $\vec{a} \times \hat{i} = \hat{j}$, find $\vec{a} \cdot \hat{i}$.

12. Write the angle between the vectors $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$.

13. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a} \cdot \vec{b} = 6$, $|\vec{a}| = 3$ and $|\vec{b}| = 4$. Write the projection of $\vec{a}$ on $\vec{b}$.

14. If $\vec{b}$ is a unit vector such that $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$, find $|\vec{a}|$.

15. If A and B are events such that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.5$, then find the value of $P(B' \cap A)$.

16. If A and B are two independent events such that $P(A) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{4}$, then find $P(B)$.

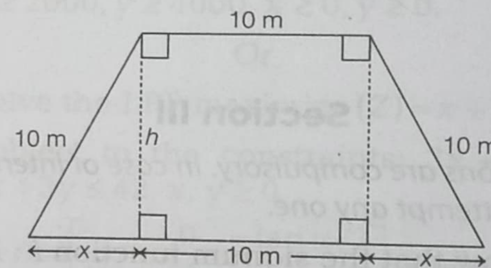
Or

If $P(A) = 0.3$, $P(B) = 0.6$, $P\left(\frac{B}{A}\right) = 0.5$, find $P(A \cup B)$.

Section II

Both the Case study based questions are compulsory. Attempt any 4 sub parts from each question. Each part carries 1 mark

17. A building has front gate has the figure as shown below.



It is in the shape of trapezium whose three sides other than base is 10 m. Height of the gate is h m.

On the basis of above figure answer the following questions.

(i) The relation between x and h is

(a) $x^2 + h^2 = 10$ (b) $x^2 + h^2 = 100$
(c) $h^2 - x^2 = 10$ (d) $x^2 - h^2 = 10$

(ii) The area of gate A expressed as a function of x is

(a) $(10 + x)\sqrt{100 + x^2}$
(b) $(10 - x)\sqrt{100 + x^2}$
(c) $(10 + x)\sqrt{100 - x^2}$
(d) $(10 - x)\sqrt{100 - x^2}$

(iii) The value of x when A is maximum, is

(a) 5 m (b) 10 m
(c) 15 m (d) 20 m

(iv) The value of h when A is maximum, is

(a) $5\sqrt{2}$ m (b) $5\sqrt{3}$ m
(c) $10\sqrt{2}$ (d) $10\sqrt{3}$ m

(v) Maximum value of A is (in m^2)

(a) $\frac{75\sqrt{3}}{2}$

(b) $75\sqrt{3}$

(c) $\frac{75\sqrt{3}}{4}$

(d) 75

18. The probability distribution function which shows the number of hours (X) a student study during lockdown period in a day, is given by

X	0	1	2
$P(X)$	$3C^3$	$4C-10C^2$	$5C-1$

Where $C>0$

On the basis of above information answer the following questions.

(i) The correct equation for C is

(a) $3C^3-10C^2+C-2=0$

(b) $3C^3+10C^2+C-2=0$

(c) $3C^3-10C^2+9C-2=0$

(d) None of the above

(ii) The value of C is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

(iii) $P(X<2)=$

(a) $\frac{1}{2}$ (b) 1/3 (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

(iv) $P(X=1)=$

(a) $\frac{2}{9}$ (b) $\frac{1}{9}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

(v) $P(X\geq 0)=$

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) 1

PART B

Section III

All questions are compulsory. In case of internal choices attempt any one.

19. Show that the signum function $f: R\to R$ given by $f(x)=\begin{cases}1, & \text{if } x>0\\ 0, & \text{if } x=0\\ -1, & \text{if } x<0\end{cases}$ is neither one-one nor onto.

Or Let $A=\{a,b,c\}$ and the relation R be defined on A as follows

$$R=\{(a,a),(b,c),(a,b)\}.$$

Then, write minimum number of ordered pairs to be added in R to make R reflexive and transitive.

20. Find the position vector of a point A in space, such that OA is inclined at 60° to OX and at 45° to OY and $|OA|=10$ units.

21. Determine the values of x for which $f(x)=\frac{x-2}{x+1}$, $x\neq -1$ is increasing or decreasing.

22. If $\sec\left(\frac{x+y}{x-y}\right)=a$, then prove that $\frac{dy}{dx}=\frac{y}{x}$.

Or

If $x\sqrt{1+y}+y\sqrt{1+x}=0$ and $x\neq y$, prove that $\frac{dy}{dx}=-\frac{1}{(x+1)^2}$.

23. If $A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$, then find the values of θ satisfying the equation $A^T+A=I$

24. Find the probability of drawing a diamond card in each of the two consecutive draws from a well-shuffled pack of cards, if the card drawn is not replaced after the first draw.

25. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular unit vectors, find $|2\vec{a}+\vec{b}+\vec{c}|$.

26. Find the value of $\int \frac{\tan^2 x \sec^2 x}{1-\tan^6 x}dx$

Or

Find the value of $\int \sin x\cdot\log\cos x\, dx$.

27. Evaluate: $\int_0^{\frac{\pi}{2}} \log \tan x \, dx.$

28. Solve: $(1+x^2)\frac{dy}{dx}+2xy=\cot x.$

Section IV

All questions are compulsory. In case of internal choices attempt any one.

29. If $\cos^{-1} x+\cos^{-1} y+\cos^{-1} z=\pi.$
Prove that $x^2+y^2+z^2+2xyz=1$

Or Find the domain of function
$f(x)=\cos^{-1}(x^2-4).$

30. Evaluate $\int(\sqrt{\tan x}+\sqrt{\cot x})\,dx.$

Or Evaluate $\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx.$

31. Solve $(x^3-3xy^2)\,dx=(y^3-3x^2y)\,dy.$

32. Find the area bounded by the curve $x^2+y^2=a^2$ using integration.

33. Find the equations of tangent and normal to the curve $y=\frac{(x-7)}{(x-2)(x-3)}$ at the point, where it cut the X-axis.

34. Show that the equation of normal at any point on the curve $x=3\cos\theta-\cos^3\theta$,
$y=3\sin\theta-\sin^3\theta$ is
$4(y\cos^3\theta-x\sin^3\theta)=3\sin 4\theta.$

35. If $y=\frac{x\cos^{-1}x}{\sqrt{1-x^2}}-\log\sqrt{1-x^2}.$

Prove that $\frac{dy}{dx}=\frac{\cos^{-1}x}{(1-x^2)^{\frac{3}{2}}}.$

Section V

All questions are compulsory. In case of internal choices attempt any one.

36. Find the coordinates of the point P where the line through $A(3,-4,-5)$ and $B(2,-3,1)$ crosses the plane passing through three points $L(2,2,1), M(3,0,1)$ and $N(4,-1,0)$. Also, find the ratio in which P divides the line segment AB.

Or

Find the distance of the point $(1,-2,3)$ from the plane $x-y+z=5$ measured parallel to the line whose direction cosines are proportional to $(2, 3, -6)$.

37. Solve the LPP, Maximise,
$Z=0.08x+0.10y$
Subject to the constraints $x+y\le 12000$, $x\ge 2000, y\ge 4000, x\ge 0, y\ge 0.$

Or

Solve the LPP maximize $(Z)=x+y$
Subject to the constraints: $3x+2y\le 48$, $2x+3y\le 42$, $x, y\ge 0.$

38. If $A=\begin{bmatrix}0 & -\tan\alpha/2\\ \tan\alpha/2 & 0\end{bmatrix}$ and I is the identity matrix of order 2, show that

$$I+A=(I-A)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}.$$

Or

Solve the following system of equations using matrix method.

$$x+y+z=7000$$
$$10x+16y+17z=110000$$
$$x-y=0$$