ANNEXURE – C

DAV PUBLIC SCHOOLS, ODISHA ZONE

NAME OF THE EXAM: HALFYEARLY, SUBJECT :MATHEMATICS,
CLASS : STD – XII

MARKING SCHEME SET- 2

QSTN NO	VALUE POINTS	MARKS ALLOTTED	PAGE NO. OF NCERT TEXT BOOK
	SECTION – A		
1	(b) $2^n - 2$	1 Mark	Exemplar
2	(c) 2 sq units	1 Mark	Exemplar
3	(b) $\frac{256}{3} sq.units$	1 Mark	Exemplar
4	(b) 2	1 Mark	Exemplar
5	(c)y	1 Mark	NCERT
6	(a) $e^x + e^{-y} = c$	1 Mark	NCERT
7	(d) 40	1 Mark	Exemplar
8	$(d) -\frac{\pi}{8}$	1 Mark	Exemplar
9	(a) $-\frac{5\pi}{12}$	1 Mark	Exemplar
10	(b) ± 15	1 Mark	NCERT
11	(c) 2	1 Mark	NCERT
12	(a) 3/4t	1 Mark	NCERT

13	(d) (0,2)	1 Mark	NCERT
14	(c) $10\sqrt{3}cm^2/\sec$	1 Mark	NCERT
15	(c) 1	1 Mark	NCERT
16	(c) x + c	1 Mark	Exemplar
17	(d)$-\log_e(1+e^{-x})+c$	1 Mark	Exemplar
18	(b)$\frac{\pi x}{2}-\frac{x^2}{2}+c$	1 Mark	NCERT
19	(d)	1 Mark	NCERT
20	(b)	1 Mark	NCERT
	SECTION – B		
21	For correct one-one proof $y=\frac{2x}{5x+3}\Rightarrow x=\frac{3y}{2-5y}$ For every $y\in R-\left\{\frac{2}{5}\right\}$, there exists $x\in R-\left\{\frac{-3}{5}\right\}$ such that $f(x)=f\left(\frac{3y}{2-5y}\right)=2\left(\frac{3y}{2-5y}\right)\div\left(5\frac{3y}{2-5y}+3\right)=y$ So, f is onto. OR As f(2) = f($\frac{1}{2}$) = $\frac{2}{5}$ So , f is not one – one Let f(x) =1 $\Rightarrow \frac{x}{1+x^2}=1$ $\Rightarrow x^2$-x +1 = 0 $\Rightarrow x \notin R$ *so* , f is not on to.	1 1 1 1	Exemplar

22	$\sin^{-1}\left[\cos\left(8\pi+\frac{3\pi}{5}\right)\right]=\sin^{-1}\left[\cos\left(\frac{3\pi}{5}\right)\right]$ $=\sin^{-1}\left[\sin\left(\frac{\pi}{2}-\frac{3\pi}{5}\right)\right]=-\frac{\pi}{10}$	1 1	NCERT
23	$\tan^{-1}\left(\frac{cosx}{1-sinx}\right)=\tan^{-1}\left(\frac{\cos^2 x/2-\sin^2 x/2}{(cosx/2-sinx/2)^2}\right)$ $=\tan^{-1}\left(\frac{1+tanx/2}{1-tanx/2}\right)=\tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)$ $=\frac{\pi}{4}+\frac{x}{2}$	1 Mark 1 Mark	Exemplar
24	For finding the correct derivative of f(x) Verifying that the function is increasing for x > -1 OR $\frac{dC}{dx}$ =3(0.005)x^2 -2(0.02) x+30 $=$ 0.15x^2 +0.04x+30 ∴ Marginal cost at x=3 is 31.47	1 1 1 1	NCERT
25	$\int\frac{x-3}{(x-1)^3}e^x dx=\int\frac{x-1-2}{(x-1)^3}e^x dx$ $=\int\left\{\frac{1}{(x-1)^2}-\frac{2}{(x-1)^3}\right\}e^x dx$ $=\frac{e^x}{(x-1)^2}+c$	1 1	NCERT
SECTION – C			
26	Using property on the given integral Simplifying the integral Finding the value of the integral	1 1 1	NCERT
27	$\frac{dy}{dx}=e^{3x}e^{4y}\Rightarrow\int e^{-4y}dy=\int e^{3x}dx$ $\Rightarrow\frac{e^{-4y}}{-4}=\frac{e^{3x}}{3}+$ c, putting x = 0 and y = 0 we get c = -7/12 Hence solution is $\frac{e^{-4y}}{-4}=\frac{e^{3x}}{3}-\frac{7}{12}$ that is $4e^{3x}+3e^{-4y}$ - 7 = 0 OR $y\,dx-(x+2y^2)\,dy=0$ $\Rightarrow y\,dx=(x+2y^2)\,dy$	1 1 1	NCERT NCERT

	$\Rightarrow \frac{dx}{dy} = \frac{(x+2y^2)}{y}$ $\Rightarrow \frac{dx}{dy} - \frac{x}{y} = 2y$ *This is a linear differential equation of the type* $\frac{dx}{dy} + P1x = Q1$ Where P1=$\frac{-1}{y}$ and Q1 = 2y Therefore IF = $e^{\int \frac{-1}{y}dy} = e^{-logy} = \frac{1}{y}$ Hence the solution of the given differential equation is $$x\,(IF) = \int Q1(IF)dy \quad + C$$ $\Rightarrow x\frac{1}{y} = \int 2y \times \frac{1}{y}dy$ + C $\Rightarrow\ x\ = 2y^2 + Cy$ *This is a general solution of the given differential*	0.5 1 0.5 1	
28	Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$ $\Rightarrow\begin{bmatrix} 2a-c & -2b-d \\ a & b \\ -3a+4c & -3b+4d \end{bmatrix} = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$ On solving $a = 1, b = -2, c = 3, d = 12$ Hence $A = \begin{bmatrix} 1 & -2 \\ 3 & 12 \end{bmatrix}$	0.5 1 0.5	NCERT - Exemplar
29	y=$(logx)^x$+$(x)^{cosx}$ y=u+v u=$(logx)^x$ and v=$(x)^{cosx}$ finding $\frac{du}{dx}$ finding $\frac{dv}{dx}$ $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$	0.5 1 1 0.5	NCERT

	OR $\begin{cases} \frac{x-4}{\|x-4\|}+a, if x<4 \\ a+b, \quad if x=4 \\ \frac{x-4}{\|x-4\|}+b, if x>4 \end{cases}$ is a continuous function at x=4. LHL = $\lim_{x\to 4^-} f(x)$= $\lim_{x\to 4^-}[\frac{(x-4)}{-(x-4)}+a]$ = $\lim_{x\to 4^-}[-1+a]$= a-1 RHL= $\lim_{x\to 4^-}[\frac{(x-4)}{\|x-4\|}+b]$ = $\lim_{x\to 4^-}[\frac{(x-4)}{(x-4)}+b]$ = $\lim_{x\to 4^-}[1+b]$=1+b f(4)=a+b As *f is continuous at x=4* *LHL=RHL =f(4)* *a-1 = a+b=1+b* *a-1=a+b&a+b=1+b* *b=-1 &a =1* *So a=1,b=-1*	1 1 1	Exemplar
30	$f(x)=20-9x-6x^2-x^3$ $\Rightarrow f'(x)=-9-12x-3x^2=-3(x+1)(x+3)$ $f'(x)=0 \Rightarrow x=-1,-3$ So f is strictly decreasing in $(-\infty,-3)\cup(-1,\infty)$ and increasing in (-3, -1). OR $f(x)=\sin x-\cos x$ $\Rightarrow f'(x)=\cos x+\cos x$ $f'(x)=0 \Rightarrow x=\frac{3\pi}{4},\frac{7\pi}{4}$ So f is strictly decreasing in $\left(\frac{3\pi}{4},\frac{7\pi}{4}\right)$ and increasing in $\left(0,\frac{3\pi}{4}\right)\cup\left(\frac{7\pi}{4},2\pi\right)$	1 Mark 1 Mark 1 Mark 1 Mark 1 Mark 1 Mark	Exemplar NCERT
31	$\int \frac{x^2}{(x^2+4).(x^2+9)}dx$ x^2=y	1	

	$\frac{x^2}{(x^2+4).(x^2+9)}=\frac{y}{(y+4).(y+9)}=\frac{A}{(y+4).}+\frac{B}{(y+9)}$	1	NCERT
	A= $\frac{-4}{5}$ B= $\frac{9}{5}$		
	$\int\frac{x^2}{(x^2+4).(x^2+9)}dx=\int\frac{\frac{-4}{5}dx}{(x^2+4).}+\int\frac{\frac{9}{5}dx}{(x^2+9)}$	1	
	$=\frac{-4}{10}tan^{-1}$x+$\frac{9}{15}tan^{-1}$x+c		
SECTION – D			
32	(LHD at x=1)= $\lim_{x\to1^-}\frac{f(x)-1}{x-1}$ = $\lim_{h\to0}\frac{h^2-5h}{-h}$ =5 Now, (RHD at x=1)= $\lim_{x\to1^+}\frac{f(x)-1}{x-1}$ = $\lim_{h\to0}\frac{bh}{h}$ =b Since, f(x) is differentiable, so (LHD at x=1)=(RHD at x=1) b=5 And f(1)=1+3+a=4+a Now, LHL= $\lim_{x\to1^-}$ f(x) LHL= $\lim_{h\to0}$(1-h)^2+3(1-h)+a LHL=1+3+a=4+a Now, RHL= $\lim_{x\to1^+}$ f(x) RHL= $\lim_{h\to0}$ b(1+h) RHL=b+2 Since, f(x) is continuous, so LHL=RHL=f(1) 4+a=b+2 4+a=5+2 a=7−4=3 Hence, a=3 and b=5.	1 1 1 1 1	Exemplar

33	Proving the properties of $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ I= $\int_0^{\pi/2} \frac{sin^2x}{sinx+cosx}$dx ………(1) I= $\int_0^{\pi/2} \frac{cos^2x}{sinx+cosx}$dx ………(2) 2I=$\int_0^{\pi/2} \frac{1}{sinx+cosx}$dx 2I=$\frac{1}{\sqrt{2}}\int_0^{\pi/2} cosec(x+\frac{\pi}{4})$ dx I= $\frac{1}{\sqrt{2}}$log($\sqrt{2}$+1) .	2 1 0.5 1.5	NCERT EXEMPLAR
34	$a, b \in N$ $\Rightarrow$ab(a+b)=ba(a+b) $\therefore$ (a, b)R(a, b) for all (a, b) ∈ N × N Hence, R is reflexive. Let (a, b), (c, d) be an arbitrary element of N × N such that (a, b)R(c, d). $\therefore$ ad(b+c)=bc(a+d) $\Rightarrow$cb(d+a)=da(c+b) $\Rightarrow$(c,d)R(a,b) $\therefore$ (a, b)R(c, d) ⇒ (c, d)R(a, b) for all (a, b), (c, d) ∈ N × N Hence, R is symmetric. ad(b+c)=bc(a+d) Also,cf(d+e)=de(c+f) $\Rightarrow$adb+adc=abc+bcd $\Rightarrow$cfd+cfe=dec+def $\Rightarrow$cd(a−b)=ab(c−d) $\Rightarrow$cd(f−e)=ef(d−c) ....(	1 1 1.5 2	NCERT-26

	$\Rightarrow aef - bef = -abf + aeb$ $\Rightarrow aef + abf = aeb + bef$ $\Rightarrow af(b+e) = be(a+f)$ $\Rightarrow (a, b)R(e, f)$ $\therefore (a, b)R(c, d)$ and $(c, d)R(e, f) \Rightarrow (a, b)R(e, f)$ for all $(a, b), (c, d), (e, f) \in N \times N$ Hence, R is transitive. Thus, R being reflexive, symmetric and transitive, is an equivalence relation on N × N. OR Here, function f: $R^+ \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$ One-one function: Let $x_1, x_2 \in R^+$ such that $f(x_1) = f(x_2)$ Then, $9x_12 + 6x_1 - 5 = 9x_2 + 6x_2 - 5$ $\Rightarrow 9(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$ $\Rightarrow 9(x_1 + x_2)(x_1 - x_2) + 6(x_1 - x_2) = 0$ $\Rightarrow (x_1 - x_2)[9(x_1 + x_2) + 6] = 0$ $\Rightarrow x_1 - x_2 = 0$ [$\because x_1, x_2 \in R_+ \therefore 9(x_1 + x_2 + 6 \neq 0)$ $\Rightarrow x_1 = x_2, \forall x_1, x_2 \in R_+$ Therefore, f(x) is one-one function. For onto: $9x^2 + 6x - 5 - y = 0$ $\Rightarrow x = \frac{-1 \pm \sqrt{y+6}}{3}$ As $x \in R^+$, so $y \geq -5$ i.e range= $[-5, \infty)$ = Co-domain. Hence f is onto.	0.5 2.5 2.5	
35	Given that, $A = \begin{vmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{vmatrix}$, To find A^{-1}. $\|A\| = \begin{vmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{vmatrix} = 1(-1-2) -2(-2)+0 =$ $-3+4=1 \neq 0$ Hence ,A^{-1} exists. Let Cij represent the cofactor of (i,j)th Element of A. Then, $C_{11} = -3$, $C_{21} = -2$, $C_{31} = -4$	1	Exemplar

	$C_{12} = 2$, $C_{22} = 1$, $C_{32} = 2$ $C_{13} = 2$, $C_{23} = 1$, C33 = 3	1	
	Adj. A = $\begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$	0.5	
	$A^{-1} = \frac{\text{Adj.A}}{\lvert A \rvert} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$	0.5	
	The given system of equations is equivalent to the matrix equation $A^T X = B \Rightarrow X = (A^T)^{-1} B$ $\Rightarrow X = (A^{-1})^T B$	0.5	
	$= \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} -30 + 16 + 14 \\ -20 + 8 + 7 \\ -40 + 16 + 21 \end{bmatrix}$	1	
	$= \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$, Hence, $x = 0, y = -5, and\ z = -3$	0.5	
	OR		NCERT TB page-139
	$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	1	
	$\Rightarrow \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$	1	
	Now, given system of equations can be written,in matrix form, as follows $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$	1	

	$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2+0+2 \\ 9+2-6 \\ 6+1-4 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$ $\Rightarrow$x=0,y=5,z=3	1 1	
	SECTION – E		
36	(i) Point of intersections are (0,2) and (3,0) Value of the given integral is 3/2 (ii) Required area= $\frac{3\pi}{2} - 1$	1 Mark 1 Mark 2 Marks	NCERT
37	(i)Let A be the 2 × 3 matrix representing the annual sales of products in two markets. $\therefore A = \begin{bmatrix} x & y & z \\ 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix}$ Let B be the column matrix representing the sale price of each unit of products x, y, z. $\therefore B = \begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$ Now, revenue = sale price × number of items sold Therefore, the revenue collected from Market I = ₹ 46000. (ii) The revenue collected from Market II = ₹ 53000. (iii) Let C be the column matrix representing cost price of each unit of products x, y, z. Then, $C = \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$ Total cost in each market is given by $AC = \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 31000 \\ 36000 \end{bmatrix}$ Now, Profit matrix = Revenue matrix - Cost matrix =	1 Mark 1	NCERT

	Therefore, the gross profit from both the markets = ₹ 15000 + ₹ 17000 = ₹ 32000 OR |A|=1000 |A| + |adj(A)|=1001000	2 1 1	
38	Let length of square piece to be cut of be x mt. Length of box is = (8 - 2x) mt. Breadth = (3 – 2x) mt. Height = x unit (i) Volume of the box V = $x(3 - 2x)(8 - 2x)$ (ii) $\frac{dv}{dx} = (3-4x)(8-2x) + (3x-2x^2)(-2)$ $= 12x^2 - 44x + 24 = 4(3x^2 - 11x + 6)$ For $\frac{dv}{dx} = 0 \Rightarrow 3x^2 - 11x + 6 = 0$ $\Rightarrow x = 3 \; or \; x = 2/3$ $x = 3$ is not possible. So $x = 2/3$. The length of square piece is 2/3 mt. (iii) For $x < \frac{2}{3}, \; \frac{dv}{dx} > 0$ For $x > \frac{2}{3}, \; \frac{dv}{dx} < 0$ As $\frac{dv}{dx}$ changes sign from +ve to –ve as x increases So volume is maximum at $x = \frac{2}{3}$. Hence Max. Volume is 280/27 m^3. OR $\frac{d^2v}{dx^2} = 4(6x-4)$ $for \; x = \frac{2}{3}, \frac{d^2v}{dx^2} = 4\left(6 \times \frac{2}{3} - 11\right) = -28 < 0$ Volume is maximum at $x = \frac{2}{3}$. Hence Max. Volume is 280/27 m^3	1 1 2 2	OS